Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Problem 400† is stated, this equation will be found to have a root $\triangle_0 = 0$, that is the absolute term of the equation will be 0; in this case therefore the equation reduces to the third degree.

By the introduction of the new unknown

$$x = \triangle_0 - \frac{c+d}{2}, \text{ and putting } h = \frac{c-d}{2}$$

the equation takes the somewhat simpler form

$$\left\{ \frac{a}{x+h} \right\}^2 + \left\{ \frac{b}{x-h} \right\}^2 = 1,$$

$$\text{or } (x^2 - h^2)^2 = a^2(x-h)^2 + b^2(x+h)^2.$$

* * *

SOLUTIONS OF PROBLEMS IN NO. 1.

In this department we will in general publish but one solution to each problem proposed, though, in some cases, when the method pursued in the solutions is essentially different, two or more solutions of the same question will be published. Credit will be given, however, in each number to all who shall have furnished correct solutions of the questions whose solutions are published in that number.

Each solution should contain sufficient detail to be comprehended by the ordinary reader who is acquainted with the elements of the branches employed in the solution. Clearness must not be sacrificed to brevity, but, other things being equal, the brevity of a solution will determine its selection for publication.

When several persons furnish essentially the same solution to a question, his name only will be placed at the head of the published solution whose notation and phraseology are adopted.

Persons sending solutions are requested to put the solution of *each question*, together with the name of the writer, on a separate piece of paper.

Solutions have been received as follows:

R. M. DeFrance solved 1 and 2; Theo. L. DeLand solved 1; Prof. A. B. Evans solved 1, 2, 3 and 4; Prof. C. Hornung solved 2; Philip Hoagland solved 2; Henry Heaton solved 1, 2, 3 and 4; Prof. E. W. Hyde solved 3; Prof. Knisely solved 1; Miss Esther W. Matthews, (State Normal School, Kirksville, Mo.), solved 1; Artemas Martin solved 1, 2, 3 and

---

†*Yates County Chronicle.*

4; O. D. Oathout solved 2; L. Regan solved 1 and 2; E. B. Seitz solved 1, 2, 3 and 4; Elias Schneider, A. M., solved 1 and 2; S. W. Salmon solved 1, 2, 3 and 4; Prof. D. W. Sensenig solved 1, 2 and 4; Walter Siverly solved 1, 2, 3 and 4, and John M. Wilt, A. M., solved 1 and 4.

1. "Find the values of x and y in the following equations:

$$a^2 x^4 + b^2 y^4 = a^2 b^2 (x + y)^2; \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$a^2 x^2 + b^2 y^2 = a^2 b^2." \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

SOLUTION BY THEO. L. DE LAND, WASHINGTON, D. C.

Divide (1) by (2), expand $(x+y)^2$, clear of fractions, cancel like terms, factor, divide by $x\,y$, substitute $a^2\,b^2$ for its value, multiply by $2\,a\,b$, and we have

$$2\,a\,b\,x\,y = -\frac{4\,a^3\,b^3}{a^2+b^2}, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Both add to, and subtract from (2), eq. (3), and we have

$$(a\,x \pm b\,y)^2 = a^2\,b^2 \mp \frac{4\,a^3\,b^3}{a^2+b^2}.$$

The upper signs give one equation and the lower signs another.

$$\text{Hence, } x = \pm \frac{b}{2\sqrt{a^2+b^2}}\left(\sqrt{(a-b)^2-2\,a\,b} \pm \sqrt{(a+b)^2+2\,a\,b}\right)$$

$$\text{and } y = \pm \frac{a}{2\sqrt{a^2+b^2}}\left(\sqrt{(a-b)^2-2\,a\,b} \mp \sqrt{(a+b)^2+2\,a\,b}\right).$$

[Prof. Knisely writes that the question he sent (No. 1) is, by mistake slightly different from what he intended to present, which, he says, "arises from the problem of a tangent to an ellipse, the tangent being intercepted between the axes produced, and equal to the sum of the semi-axes." The question, as he intended to present it, and his solution are subjoined:]

$$\text{"}a^2 x^2 + b^2 y^2 = x^2 y^2. \quad . \; . \; . \; . \; . \; . \quad (1)$$

$$b^2 y^4 + a^2 x^4 = a^2 b^2 (x + y)^2. \quad . \; . \; . \; . \quad (2)$$

SOLUTION.

$$\text{From (1) } y^2 = \frac{a^2 x^2}{x^2 - b^2}, \quad y = \frac{a\,x}{\sqrt{x^2 - b^2}}.$$

Hence, in (2),

$$a^2 x^4 + \frac{a^4 b^2 x^4}{(x^2 - b^2)^2} = a^2 b^2\left(x^2 + \frac{2\,a\,x^2}{\sqrt{x^2 - b^2}} + \frac{a^2 x^2}{x^2 - b^2}\right).$$

Dividing by $a^2 x^2$,

$$x^2 + \frac{a^2 b^2 x^2}{(x^2 - b^2)^2} = b^2 + \frac{2\,a\,b^2}{\sqrt{x^2 - b^2}} + \frac{a^2 b^2}{x^2 - b^2}.$$

$$x^2 (x^2 - b^2)^2 + a^2 b^2 x^2 = (x^2 - b^2)^2 b^2 + 2\,a\,b^2 (\sqrt{x^2 - b^2})^3 + (x^2 - b^2)\,a^2 b^2.$$

$$x^6 - 2\,x^4\,b^2 + b^4\,x^2 + a^2\,b^2\,x^2 = b^2\,x^4 - 2\,b^4\,x^2 + b^6 + 2\,a\,b^2\,(\sqrt{x^2 - b^2})^3 + a^2\,b^2\,x^2 - a^2\,b^4$$

$$x^6 - 3\,x^4\,b^2 + 3\,x^2\,b^4 - b^6 - 2\,a\,b^2\,(\sqrt{x^2 - b^2})^3 + a^2\,b^4 = 0.$$

$$(x^2 - b^2)^3 - 2\,a\,b^2\,(x^2 - b^2)^{\frac{3}{2}} + a^2\,b^4 = 0.$$

Taking square root,

$$\sqrt{(x^2 - b^2)^3} = a\,b^2. \quad x^2 = (a\,b^2)^{\frac{2}{3}} + b^2. \quad y^2 = (a^2\,b)^{\frac{2}{3}} + a^2.$$

—U. Jesse Knisely."

---

2. "Let a regular polygon of 14 sides be described, each of whose equal sides shall be *one*. Then will the radius of its circumscribing circle, which put$=r$, be more than *two* and less than *three*. Put $r = 2 + x$; then is x a positive quantity less than *one*. Let another regular polygon of half the number of sides (7) be inscribed in a circle whose radius is *one*, and determine one of its equal sides in functions of x expressed in its simplest form."

SOLUTION BY PROF. ASHER B. EVANS, LOCKPORT, N. Y.

Let $A\,B$ represent a side of the regular polygon of 14 sides, and O the center of its circumscribing circle. Then

$$A\,B = 1 : A\,O = r :: \sin \tfrac{1}{7}\pi : \sin \tfrac{3}{7}\pi;$$

$$\therefore A\,O = r = \frac{\sin \frac{3}{7}\pi}{\sin \frac{1}{7}\pi} = 3 - 4 \sin^2 \tfrac{1}{7}\pi \quad . \; . \; . \quad (1)$$

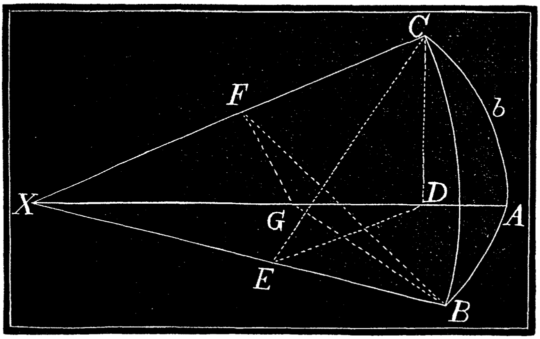
Since $2 \sin \frac{1}{7}\pi$ is less than $2 \sin 30°$, that is, less than unity, $4 \sin^2 \frac{1}{7}\pi$ is positive and less than unity, and equation (1) gives $r > 2$ and < 3.

If $r = 2 + x$, $x = 1 - 4 \sin^2 \frac{1}{7}\pi$, or $2 \sin \frac{1}{7}\pi = \sqrt{1 - x}$. But $2 \sin \frac{1}{7}\pi$ is the expression for a side of a regular polygon of 7 sides the radius of the circumscribing circle being unity; therefore the required function of x is $\sqrt{1 - x}$.

---

3. "If a line make an angle of 40° with a fixed plane, and a plane embracing this line be perpendicular to the fixed plane, how many degrees from its first position must the plane embracing the line revolve in order that it may make an angle of 45° with the fixed plane?"

SOLUTION BY PROF. E. W. HYDE, CHESTER, PA.



Let b = angle between line and plane; C = angle through which plane through line must revolve, and B = angle plane makes with fixed plane; then we have at once, by Napier's formulæ,

$$\cos B = \cos b \sin C;$$

$$\therefore \sin C = \frac{\cos B}{\cos b},$$

and if $b = 40°$ and $B = 45°$,

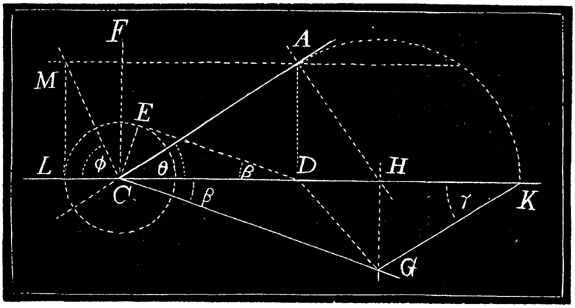
$$\sin C = \frac{\cos 45^\circ}{\cos 40^\circ} = \frac{1}{\sqrt{2}\cos 40^\circ};$$

$$\therefore C = 67^\circ\ 22'\ 40''.$$

[*All* the solutions of No. 3 which have been received, except Mr. Salmon's, are analogous to the above.]

SOLUTION BY S. W. SALMON, MOUNT OLIVE, N. J.

Take the fixed plane as the horizontal plane of projection (H). Let the given line CA make an angle θ with H, and take the vertical plane through this line as the vertical plane of projection (V). Let C be the point in which CA pierces H. Let γ be the angle through which the vertical plane through CA has to revolve in order to make an angle ϕ with H, and let β be the angle which the horizontal trace of this plan makes with the ground line. Draw CF perpendicular to H, and let it be the axis of a cone with a circular right section whose vertex is C and whose elements make an angle ϕ with H. Through CA pass a plane tangent to this cone; the tangent plane will then make an angle ϕ with H; CA is the vertical trace of this plane. In order to find the horizontal trace pass a plane parallel to H through A; it cuts a circle from the cone and a line from the plane tangent to the circle. ED is the horizontal projection of this line, and CG, drawn parallel to ED, is the horizontal trace of the plane. Through A pass a plane perpendicular to CA, AH is its vertical and GH its horizontal trace; it cuts a line from the plane GCA, the position of which when revolved around GH into the horizontal plane is GK. The angle $GKH = \gamma$.



$$LM = AD = CD \text{ tang } \theta$$

$$\text{and } LC = CE = \frac{LM}{\text{tang } \phi} = \frac{CD \text{ tang } \theta}{\text{tang } \phi} = CD \sin \beta,$$

$$\text{whence } \beta = \sin^{-1} \frac{\text{tang } \theta}{\text{tang } \phi},$$

$$\text{tang } \gamma = \frac{GH}{HK} = \frac{CH \text{ tang } \beta}{CH \sin \theta} = \text{tang} \frac{\left(\sin^{-1} \frac{\text{tang } \theta}{\text{tang } \phi}\right)}{\sin \theta}.$$

$$\therefore \gamma = \text{tang}^{-1}\left[\frac{\text{tang}\left(\sin^{-1}\dfrac{\text{tang}\,\theta}{\text{tang}\,\phi}\right)}{\sin\theta}\right].$$

When $\theta = 40°$ and $\phi = 45$, $\gamma = 67° \; 22' \; 41''$.

[This question admits of still another solution, as follows: In the figure to Prof. Hyde's solution, above, draw $C\,D$ perpendicular to $A\ X$ and $D\ E$ perpendicular to $B\,X$. Then, because the $< C\,E\,D = 45°$, $D\ E = C\,D$. $\therefore$ (if $C\,X = 1$) $C\,E = \sqrt{2}\sin 40°$, and $E\,X = \sqrt{1-2\sin^2 40°}$. Make $F\,X = E\ X$; then is $F\ G = \tan 40° \sqrt{1-2\sin^2 40°}$. Join $B\ G$ and $B\ F$. Then is $B\ G\ F$ a right angbled triangle, right angle at G, and $B\,F = C\,E$. $\therefore$ we have $B\,F : F\,G ::$ radius : cos of the requirad angle, or, $\sqrt{2}\sin 40° : \tan 40° \sqrt{1-2\sin^2 40°} :: 1 : \cos G\ F\ B = 67° \; 22' \; 41''$, very nearly.]

---

4. "A cask containing a gallons of wine stands on another containing a gallons of water; they are connected by a pipe, through which, when open, the wine can escape into the lower cask at the rate of c gallons per minute, and through a pipe in the lower cask the mixture can escape at the same rate; also, water can be let in through a pipe on the top of the upper cask at a like rate. If all the pipes be opened at the same instant, how much *wine* will be in the lower cask at the end of t minutes, supposing the fluids to mingle perfectly?"

SOLUTION BY S. W. SALMON, MOUNT OLIVE, N. J.

Let Q be the quantity of wine in the lower cask at the end of t minutes; and let Q_1, Q_2, Q_3 .... Q_n be the quantity at the end of the 1st 2d, 3d .... nth instants, q, the quantity of the mixture that escapes in an instant, and n, the number of instants in a minute, then is $n\,q = c$,

$$Q_1 = q = q\left(1 - \frac{q}{a}\right)^0, \quad Q_2 = 2\,q\left(1 - \frac{q}{a}\right),$$

$$Q_3 = 3q\left(1 - \frac{q}{a}\right)^2 \;\ldots\; Q_n = n\,q\left(1 - \frac{q}{a}\right)^{n-1}.$$

$$\therefore Q = t\,n\,q\left(1 - \frac{q}{a}\right)^{t\,n-1}.$$

Taking the Napierian logarithm of both members of this equation,

$$\log Q = t\,n \log\left(1 - \frac{q}{a}\right) + \log c\,t$$

$$= \log c\,t + t\,n\left(-\frac{q}{a} - \frac{q^2}{2\,a^2} - \frac{q^3}{3\,a^3} - \&c.\right).$$

Neglecting powers of infinitesimals, we have

$$\log\left(\frac{c\,t}{Q}\right) = \frac{t\,n\,q}{a} = \frac{c\,t}{a}.$$

$$\therefore e^{\frac{c\,t}{a}} = \frac{c\,t}{Q}. \quad \therefore Q = c\,t\,e^{-\frac{c\,t}{a}};$$

where e denotes the base of the Napierian system of logarithms.

[This question was solved by Prof. Evans in an elegant manner by the method of finite differences, and in nearly the same manner as the above by Prof. Sensenig. All the other solutions were by application of the differential and integral calculus.]

QUERY.—Can a demonstration be given of the following formula for primes?

$$N = \frac{x}{A \log x - B};$$

in which N denotes the number of prime numbers contained in any number x, and A and B are constants.—Communicated by PROF. EDWARD BROOKS.

NOTE ON SUN SPOTS.—One might easily gather from reading astronomical works, that solar spots are rarely visible to the naked eye; that is, without the use of a telescope; but such is not the case. Let any one who feels an interest in the subject, prepare a suitable smoked glass, and examine the sun's disk daily, or as often as the clouds will permit, and he will find that solar spots can frequently be seen without a telescope. If it were worth the space to record the observations, I could give numerous instances when I saw spots without a telescope, and I have occasionally seen two at a time.

Since the sun's spots return periodically once in about eleven years; that is, from the minimum average number they gradually increase in number and area till the maximum is reached in about five and a half years, when the number gradually decreases; large spots are more likely to be seen about the time of the maximum number.

To discover a solar spot without a telescope, keep the eye directed atten-